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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2015

MA101 CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer ALL Questions

1. Find the derivative of tanh$\sqrt{1+x^2}$ 2
2. Examine the convergence of the series $\sum_{k=1}^{\infty}\frac{k^k}{k!}$ 3
3. Convert the rectangular coordinates $(0,4,\sqrt{3})$ to cylindrical and spherical coordinates 2
4. Find equations of the paraboloid $z^2 = x^2 + y^2$ in cylindrical and spherical coordinates 3
5. If $U=\frac{x^3+y^3}{x-y}$, . Find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}$ 2
6. The length, width, and height of a rectangular box are measured with an error of at most 5%. Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box. 3
7. Find $\nabla z, if, z = 4x-10y$. 2
8. A particle moves on the curves x=$2t^2$, y=t^2-4t,z=3t-5 where t is the time .Find the component of acceleration at the time t=1 in the direction$\hat{\imath}$ -3$\hat{\jmath}$ +2$\hat{k}$. 3
9. Evaluate$\int_0^1\int_{x^2}^{2-x} xydydx$ 2
10. Find the Jacobian of the transformations x = uv and y = $\frac{u}{v}$ 3
11. Find curl $\vec{F}$ at the point (1,-1,1) where $\vec{F}=xz^3\hat{\imath}-2x^2yz\hat{\jmath}+2yz^4\hat{k}$ 2
12. The function ϕ(x,y,z) = xy+yz+xz is a potential for the vector field $\vec{F}$, find the vector field $\vec{F}$. 3

PART B

MODULE 1

Answer ANY TWO Questions

13. Find the Maclaurin series for cosx and also find cos 1,calculate the absolute error 5

14. Prove that $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$,-1< x< 1 5

15. Show the series $\sum_{k=1}^{\infty}(\frac{1}{2})^k$ converges and$\sum_{k=1}^{\infty}(-1)^k$ diverges 5

MODULE 2

Answer ANY TWO Questions

16. Find the natural domain of the following functions.
    i. $f(x,y) = 3x^2\sqrt{y} - 1$
    ii. $f(x,y) = \log(x^2 - y)$ 5

17. Evaluate $\underset{(x,y)\to(-1,2)}{Lt} \frac{x^2+y}{x^2+y^2}$. State the properties used in the evaluation. 5

18. Find the traces of the surface $x^2 + y^2 - z^2 = 0$ in the planes x=2 and y=1 and identify the same. 5

MODULE 3

Answer ANY TWO Questions

19. Find maximum and minimum values of
    $f(x, y)=x^3+3xy^2-15x^2-15y^2+72x$ 5

20. Let L(x, y,z) denote the local linear approximation to $f(x,y,z) = \frac{x+y}{y+z}$ at the point P(-1,1,1). Compare the error in approximating f by L at Q(-0.99,0.99, 0.01)with the distance between P and Q . 5

21. $z = 3xy^2z^3$; $y = 3x^2 + 2$; $z = \sqrt{x-1}$ Find $\frac{dw}{dx}$ and $\frac{dw}{dy}$ 5

MODOULE 4

Answer ANY TWO Questions

22. Given a circular helix r(t)=acost$\hat{\imath} + asint\hat{\jmath} + bt\hat{k}$,a, b>0 , $0 \le t \le \infty$,find its arc length and unit tangent vector. 5
23. The position vector at any time t of a particle moving along a curve is $\vec{r}$(t)=t$\hat{\imath}$ + $t^2\hat{\jmath}$+$t^3\hat{k}$.
    Find the scalar and vector tangential and normal component of the acceleration at time t=1 5
24. Find the parametric equation of the tangent line to the curve of intersection of the paraboloid $z = x^2$+$y^2$and the ellipsoid $3x^2 + 2y^2 + z^2 = 9\ at\ (1,1,2)$ 5

MODULE 5

Answer ANY THREE Questions

25. Evaluate$\iint (x^2 + y^2)dxdy$ over the region in the positive quadrant for which x+ y $\le$ 1 5
26. Change the order of integration in$\int_0^1 \int_x^1 \frac{x}{x^2+y^2} dxdy$ and hence evaluate the same. 5
27. Find the area bounded by the Parabolas y^2=4xand x^2=-(y/2). 5
28. Find the volume bounded by the cylinder $x^2+y^2=4$ the planes y+ z=3 and z=0 5
29. Evaluate$\int_0^\infty \int_0^\infty e^{-(x+y)} \sin\left(\frac{\pi y}{x+y}\right) dxdy$
    by means of the transformation u = x+y, v=y 5

MODULE 6

Answer ANY THREE Questions

30. Use Green's theorem to evaluate $\oint_C (xcosydx - ysinxdy)$ where C is the square with vertices (0, 0), (π, 0), (π, π) and (0, π) 5
31. Use Stoke's theorem to evaluate the integral $\oint_c \vec{F}.dr$, where $\vec{F} = xy\,\hat{\imath} + yz\,\hat{\jmath} + zx\,\hat{k}$; C is the triangle in the plane x+y+z = 1 with vertices (1,0,0) ,(0,1,0) and (0,0,1)with a counter clockwise orientation looking from the first octant towards the origin. 5

32. Use Gauss Divergence Theorem to find the outward flux of vector field $\vec{F}(x,y,z) = x^3 i + y^3 j + z^3 k$ across the surface of the region enclosed by circular cylinder $x^2 + y^2 = 9$ and the plane $z = 0$ and $z = 2$ 5

33. Use Gauss Divergence Theorem to find the outward flux of vector field $\vec{F}(x,y,z) = x^3 i + y^3 j + z^3 k$ across the surface of the region enclosed by circular cylinder $x^2 + y^2 = 9$ and the plane $z = 0$ and $z = 2$ 5

34. Find the work done by the force field $\vec{F}(x,y) = (e^x - y^3)\hat{\imath} + (\cos y + x^3)\hat{\jmath}$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counter clockwise direction. 5