

Reg. No. _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SECOND SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

MA 102: DIFFERENTIAL EQUATIONS

Max. Marks: 100

Duration: 3Hours

PART A

Answer all questions. 3 marks each.

1. Solve the initial value problem $y'' - y = 0$, $y(0) = 4$, $y'(0) = -2$
2. Show that e^{2x} , e^{3x} are linearly independent solutions of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ in $-\infty < x < +\infty$. What is its general solution?
3. Solve $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$
4. Find the particular integral of $(D^2 + 4D + 1)y = e^x \sin 3x$
5. Find the Fourier series of $f(x) = x$, $-\pi \leq x \leq \pi$
6. Obtain the half range cosine series of $f(x) = x^2$, $0 \leq x \leq C$
7. Form the partial differential equation from $z = xg(y) + yf(x)$
8. Solve $(y - z)p + (x - y)q = (z - x)$
9. Write down the important assumption when derive one dimensional wave equation.
10. Solve $3u_x + 2u_y = 0$ with $u(x,0) = 4e^{-x}$ by the method of separation of variables.
11. Solve one dimensional heat equation when $k > 0$
12. Write down the possible solutions of one dimensional heat equation.

PART B

Answer six questions, one full question from each module.

Module I

13. a) Solve the initial value problem $y'' - 4y' + 13y = 0$ with $y(0) = -1$, $y'(0) = 2$ (6)
- b) Solve the boundary value problem $y'' - 10y' + 25y = 0$, $y(0) = 1$, $y(1) = 0$ (5)

OR

14. a) Show that $y_1(x) = e^{-4x}$ and $y_2(x) = xe^{-4x}$ are solutions of the differential equation $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$. Are they linearly independent? (6)
- b) Find the general solution of $(D^4 + 3D^2 - 4)y = 0$. (5)

Module II

15. a) Solve $(D^3 + 8)y = \sin x \cos x + e^{-2x}$ (6)
- b) Solve $y'' + y = \tan x$ by the method of variation of parameters. (5)

OR

16. a) Solve $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = \frac{1}{x}$ (6)

b) Solve $(D^2 + 2D - 3)y = e^x \cos x$ (5)

Module III

17. a) Find the Fourier series of $f(x) = \begin{cases} -1 + x, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{cases}$ (6)

b) Find the half range sine series of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$ (5)

OR

18. a) Obtain the Fourier series of $f(x) = \begin{cases} -\frac{\pi}{4}, & -\pi < x < 0 \\ \pi/4, & 0 < x < \pi \end{cases}$ (6)

b) Find the half range cosine series of $f(x) = x, 0 < x < l$ (5)

Module IV

19. a) Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$ (6)

b) Find the Particular Integral of $\frac{\partial^3 z}{\partial x^3} - 7 \frac{\partial^3 z}{\partial x^2 \partial y} - 6 \frac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$ (5)

OR

20. a) Solve $(D^2 + DD' - 6D'^2)z = y \sin x$ (6)

b) Solve $(mz - ny)p + (nx - lz)q = ly - mx$ (5)

Module V

21. Solve the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0, t) = 0, u(l, t) = 0$ for all t and initial conditions $u(x, 0) = f(x), \frac{\partial u}{\partial t}\bigg|_{t=0} = g(x)$. (10)

OR

22. A string of length 20cm fixed at both ends is displaced from its position of equilibrium, by each of its points an initial velocity given by $= \begin{cases} x, & 0 < x \leq 10 \\ 20 - x, & 10 \leq x \leq 20 \end{cases}$, x being the distance from one end. Determine the displacement at any subsequent time. (10)

Module VI

23. Derive one-dimensional heat equation. (10)

OR

24. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0°C , assuming that the initial temperature is $f(x) = \begin{cases} x, & 0 < x < L/2 \\ L - x, & L/2 < x < L \end{cases}$ (10)
